Block Trigonometric Transform Adaptive Filtering*

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Abstract

Frequency-domain implementations improve the computational efficiency and the convergence rate of adaptive schemes. This paper develops frequency-domain adaptive structures that are based on the trigonometric transforms DCT *and* DST. *The structures involve only real arithmetic and efficient algorithms exist for computing these transforms. The new filters are derived by first presenting a derivation for the classical* DFT*-based filter that allows us to pursue these extensions very immediately.*

1 Introduction

Computational complexity is a burden in applications that require long tapped-delay adaptive structures, such as acoustic echo cancelation where filters with hundreds or even thousands of taps are necessary to model the echo path. Frequency-domain and subband adaptive filters have been proposed to reduce the computational requirements inherent to such applications (see, e.g., [1, 2, 3, 4]). These techniques not only result in more efficient computations (due to the use of efficient FFT implementations and block signal processing) but they also improve the convergence rate of an adaptive algorithm (due to a decrease in the eigenvalue spread of the correlation matrix of the signals in the subbands). It is also known that the DFT matrix uncorrelates stationary signal vectors whose covariance matrices are circulant so that, in this case, the input signals to the subband filters will be further uncorrelated.

In this paper we develop frequency-domain adaptive structures that are based on the trigonometric transforms DCT and DST. The resulting filters will involve only real arithmetic and efficient algorithms exist for computing the DCT and DST (see, e.g., [5]). They are also better suited to input signals whose covariance matrices can be diagonalized by the DCT and DST (e.g., covariance matrices that can be represented as the sum of Toeplitz and Hankel matrices – see Sec. 4). The new adaptive structures are derived by first presenting a derivation for the classical DFT-based filter that allows us to pursue these extensions very immediately.

2 A Block Estimation Problem

Consider two jointly wide-sense stationary (WSS) and zero-mean random sequences $\{x(n), d(n)\}$, and define their $M-$long block column versions

$$\begin{aligned}\mathbf{x}(n) &= \text{col}\{x(Mn), x(Mn-1), \ldots, x(Mn-M+1)\}\\ \mathbf{d}(n) &= \text{col}\{d(Mn), d(Mn-1), \ldots, d(Mn-M+1)\}\end{aligned}$$

The $z-$spectrum of $\{\mathbf{x}(n)\}$ and the cross z-spectrum of $\{\mathbf{d}(n)\}$ and $\{\mathbf{x}(n)\}$ are denoted by $\mathbf{S_x}(z)$ and $\mathbf{S_{dx}}(z)$, respectively. The linear least-mean-squares filter for estimating $\mathbf{d}(n)$ from $\{\mathbf{x}(n), -\infty < n < \infty\}$ is given by $\mathbf{G}(z) = \mathbf{S_{dx}}(z)\mathbf{S_x}^{-1}(z)$, and is represented in Fig. 1. The signal $\mathbf{e}(n)$ denotes the estimation error, $\mathbf{e}(n) = \mathbf{d}(n) - \hat{\mathbf{d}}(n)$, and, using $z-$transform notation, $\hat{\mathbf{d}}(z) = \mathbf{G}(z)\mathbf{x}(z)$.

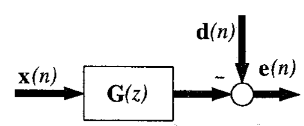

Figure 1. Block estimation problem.

It is known that for WSS signals $\{x(n), d(n)\}$ the spectra $\mathbf{S_x}(z)$ and $\mathbf{S_{dx}}(z)$ are pseudocirculant (PC) matrix[1] functions [6]. Moreover, since the inverse of a PC matrix is also PC, and since the product of two PC matrices is PC,

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[1] A pseudocirculant matrix function $\mathbf{K}(z)$ is essentially a circulant matrix function with the exception that all the entries below the main diagonal are further multiplied by z^{-1} — see Eq. (1).

it follows that the optimal filter $\mathbf{G}(z)$ is also PC, viz., it has the form (for $M = 3$)

$$\mathbf{G}(z) = \begin{bmatrix} g_0(z) & g_1(z) & g_2(z) \\ z^{-1}g_2(z) & g_0(z) & g_1(z) \\ z^{-1}g_1(z) & z^{-1}g_2(z) & g_0(z) \end{bmatrix} . \quad (1)$$

[In fact, the entries $g_i(z)$ represent the polyphase components of the (wideband) LTI filter $G(z)$ that estimates $d(n)$ from $\{x(n),\ -\infty < n < \infty\}$.]

Due to its structure, the matrix $\mathbf{G}(z)$ can be factored into $\mathbf{G}(z) = \mathbf{P}(z)\mathbf{Q}(z)$, where $\mathbf{P}(z)$ is an $M \times (2M-1)$ matrix function with Toeplitz structure, e.g., for $M = 3$,

$$\mathbf{P}(z) = \begin{bmatrix} g_0(z) & g_1(z) & g_2(z) & 0 & 0 \\ 0 & g_0(z) & g_1(z) & g_2(z) & 0 \\ 0 & 0 & g_0(z) & g_1(z) & g_2(z) \end{bmatrix} , \quad (2)$$

and $\mathbf{Q}(z)$ is a $2M-1 \times M$ matrix with a leading identity block and a lower block with shifts, say for $M = 3$ again,

$$\mathbf{Q}(z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ z^{-1} & 0 & 0 \\ 0 & z^{-1} & 0 \end{bmatrix} . \quad (3)$$

3 The DFT-Based Adaptive Structure

The pseudocirculant structure of $\mathbf{G}(z)$ can be exploited to derive a known frequency-domain adaptive filter that relies on the DFT, and which is known in the literature as the multidelay adaptive filter (or MDF – see [1, 2, 3]). The original derivation of this structure is considerably different from the approach we present in this paper. Our derivation is based on exploiting in a direct way the PC structure of $\mathbf{G}(z)$. As a fallout, the argument will suggest immediate extensions that rely on other signal transformations (such as the real trigonometric transforms DCT and DST – see Sec. 4).

We start by embedding the $M \times (2M-1)$ Toeplitz matrix $\mathbf{P}(z)$ into a $(2M-1) \times (2M-1)$ circulant matrix function $\mathbf{C}(z)$ (a similar technique was used in [7] to propose efficient structures for block digital filtering), say for $M = 3$,

$$\mathbf{C}(z) = \left[\begin{array}{ccccc} g_0(z) & g_1(z) & g_2(z) & 0 & 0 \\ 0 & g_0(z) & g_1(z) & g_2(z) & 0 \\ 0 & 0 & g_0(z) & g_1(z) & g_2(z) \\ \hline g_2(z) & 0 & 0 & g_0(z) & g_1(z) \\ g_1(z) & g_2(z) & 0 & 0 & g_0(z) \end{array}\right] , \quad (4)$$

so that $\mathbf{P}(z) = [\mathbf{I}_M \quad \mathbf{0}]\mathbf{C}(z)$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix and $\mathbf{0}$ is the $M \times M-1$ null matrix. Now the circulant matrix $\mathbf{C}(z)$ can be diagonalized by the $(2M-1) \times (2M-1)$ DFT matrix $\mathbf{F}$ as $\mathbf{C}(z) = \mathbf{F}^*\mathbf{W}(z)\mathbf{F}$, where $\mathbf{W}(z) = \mathrm{diag}\{w_0(z), \ldots, w_{2M-2}(z)\}$ and $*$ denotes complex conjugate transposition. Note in particular that the entries of the first row of $\mathbf{C}(z)$ satisfy (e.g., for $M = 3$)

$$\begin{bmatrix} g_0(z) \\ g_1(z) \\ g_2(z) \\ 0 \\ 0 \end{bmatrix} = \mathbf{F} \begin{bmatrix} w_0(z) \\ w_1(z) \\ w_2(z) \\ w_3(z) \\ w_4(z) \end{bmatrix} . \quad (5)$$

In other words, not every diagonal matrix $\mathbf{W}(z)$ in $\mathbf{C}(z) = \mathbf{F}^*\mathbf{W}(z)\mathbf{F}$ will result in a circulant matrix $\mathbf{C}(z)$ of the form (4). This is because the transformation (5) shows that the $\{w_i(z)\}$ should be such that the last two entries of the transformed vector are zero. We shall invoke this constraint at the end of this section when deriving the so-called *constrained* adaptive structure.

We can now write $\mathbf{G}(z) = \mathbf{P}(z)\mathbf{Q}(z)$ in the form

$$\mathbf{G}(z) = [\mathbf{I}_M \quad \mathbf{0}]\mathbf{F}^*\mathbf{W}(z)\mathbf{F}\mathbf{Q}(z) . \quad (6)$$

The estimation error $\mathbf{e}(n) = \mathbf{d}(n) - \hat{\mathbf{d}}(n)$ is then given by, in the z-transform domain,

$$\mathbf{e}(z) \;=\; \mathbf{d}(z) - [\mathbf{I}_M \quad \mathbf{0}]\mathbf{F}^*\mathbf{W}(z)\mathbf{F}\mathbf{Q}(z)\mathbf{x}(z) . \quad (7)$$

Now define the $(2M-1) \times 1$ transformed signal

$$\mathbf{x}'(z) \triangleq \mathbf{F}\mathbf{Q}(z)\mathbf{x}(z) \triangleq \mathrm{col}\{x_0'(z), x_1'(z), \ldots, x_{2M-2}'(z)\}.$$

Then

$$\mathbf{W}(z)\mathbf{x}'(z) = \mathrm{col}\{w_0(z)x_0'(z), \ldots, w_{2M-2}(z)x_{2M-2}'(z)\} .$$

In a tapped-delay-line adaptive estimation of the filter $\mathbf{G}(z)$, we first approximate the diagonal components $\{w_i(z)\}$ by FIR filters with (column vector) weights denoted by $\{\mathbf{w}_i\}$ and of length N/M each (assuming further that the wideband filter $G(z)$ is approximated by an FIR filter of length N). In this case, the output of each term $w_i(z)x_i'(z)$ at a certain time instant n can be obtained as the inner product $\mathbf{x}_i'(n)\mathbf{w}_i$, where $\mathbf{x}_i'(n)$ is the state (row) vector corresponding to $\mathbf{w}_i$ at time n and is given by

$$\mathbf{x}_i'(n) = \begin{bmatrix} x_i'(n) & x_i'(n-1) & \ldots & x_i'\left(n - \frac{N}{M} + 1\right) \end{bmatrix} .$$

Define the $(2M-1) \times \frac{N}{M}(2M-1)$ block diagonal matrix of regression vectors at time n,

$$\overline{\mathbf{X}}(n) = \mathrm{diag}\{\mathbf{x}_0'(n), \mathbf{x}_1'(n), \ldots, \mathbf{x}_{2M-2}'(n)\} ,$$

and the following $(2M-1)\frac{N}{M}$ column vector of unknown weight vectors that we wish to determine,

$$\overline{\mathcal{W}} = \mathrm{col}\{\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_{2M-2}\} .$$

It then follows from the error equation (7) that

$$\mathbf{e}(n) = \mathbf{d}(n) - [\mathbf{I}_M \quad \mathbf{0}]\mathbf{F}^*\overline{\mathbf{X}}(n)\overline{\mathcal{W}} .$$

An LMS-based adaptive algorithm that recursively estimates the $\overline{\mathcal{W}}$ is given by

$$\overline{\mathcal{W}}_{n+1} = \overline{\mathcal{W}}_n + \mu\overline{\mathbf{X}}^*(n)\mathbf{e}'(n) , \tag{8}$$

where we introduced the transformed error signal

$$\mathbf{e}'(n) \triangleq \mathbf{F}\begin{bmatrix} \mathbf{I}_M \\ \mathbf{0} \end{bmatrix}\mathbf{e}(n). \tag{9}$$

Note in particular that the update for the estimate of the $i-$th weight vector $\mathbf{w}_i$ is of the form (in terms of the $i-$th entry of $\mathbf{e}'(n)$ and the $i-$th regression vector $\mathbf{x}_i'(n)$):

$$\mathbf{w}_{i,n+1} = \mathbf{w}_{i,n} + \mu \cdot [\mathbf{x}_i'(n)]^*\mathbf{e}_i'(n) .$$

This suggests an alternative way for rewriting the adaptive algorithm (8), where instead of collecting all unknown column weight vectors $\{\mathbf{w}_i\}$ into a single column vector $\overline{\mathcal{W}}$, we collect their conjugate transposes into a block matrix of dimensions $(2M-1) \times \frac{N}{M}$. Thus define

$$\mathcal{W} = \begin{bmatrix} \mathbf{w}_0^* \\ \mathbf{w}_1^* \\ \vdots \\ \mathbf{w}_{2M-1}^* \end{bmatrix} , \quad \mathbf{X}(n) = \begin{bmatrix} \mathbf{x}_0'(n) \\ \mathbf{x}_1'(n) \\ \vdots \\ \mathbf{x}_{2M-1}'(n) \end{bmatrix} ,$$

and $\mathbf{E}(n) = \text{diag}\{\mathbf{e}_0'(n), \ldots , \mathbf{e}_{2M-2}'(n)\}$. Then the unconstrained frequency-domain adaptive filter becomes

$$\mathcal{W}_{n+1} = \mathcal{W}_n + \mu\mathbf{\Lambda}^{-1}(n)\mathbf{E}^*(n)\mathbf{X}(n) . \tag{10}$$

where we further introduced a $(2M-1) \times (2M-1)$ diagonal weighting matrix $\mathbf{\Lambda}(n)$; its entries consist of power estimates of the inputs of the individual subband channels,

$$\mathbf{\Lambda}(n) = \text{diag}\{\lambda_0(n), \ldots , \lambda_{2M-2}(n)\} ,$$

with each $\lambda_i(n)$ evaluated via

$$\lambda_i(n) = \beta\lambda_{i-1}(n) + (1-\beta)|x_i'(n)|^2 , \quad 0 < \beta < 1 ,$$

with initial condition equal to 1.

The reason for the qualification *unconstrained* is that the filters $w_i(z)$ that result from the weight estimates in $\mathcal{W}_n$ do not necessarily satisfy the constraint (5). A *constrained* version of the algorithm is obtained as follows (as suggested by the relation (5)). We first multiply $\mathcal{W}_n$ by $\mathbf{F}$ followed by $(\mathbf{I}_M \oplus \mathbf{0})$ in order to zero out its last $M-1$ rows. We then return to the frequency domain by multiplying the result by $\mathbf{F}^*$. That is, the constrained estimate, denoted by $\mathcal{W}_n^c$, is obtained via $\mathcal{W}_n^c = \mathbf{F}^*(\mathbf{I}_M \oplus \mathbf{0})\mathbf{F}\mathcal{W}_n$, so that the recursion for the *constrained* frequency-domain adaptive filter is

$$\mathcal{W}_{n+1}^c = \mathcal{W}_n^c + \mu\mathbf{F}^*\mathbf{U}_F\mathbf{F}\mathbf{\Lambda}^{-1}(n)\mathbf{E}^*(n)\mathbf{X}(n) , \tag{11}$$

with $\mathbf{U}_F = (\mathbf{I}_M \oplus \mathbf{0})$.

4 A DCT-Based Adaptive Structure

The well-known DFT-based adaptive structure was thus rederived above by embedding the matrix $\mathbf{P}(z)$ into a larger circulant matrix $\mathbf{C}(z)$, which was then diagonalized by the DFT matrix. Now one could embed $\mathbf{P}(z)$ into other larger matrices that are not necessarily circulant, but which could still be diagonalized by other orthogonal transforms, say by trigonometric transforms. In this section, we focus on the DCT transform and, in particular, consider the following so-called DCT-III matrix, say of dimensions $K \times K$,[2]

$$\mathcal{C}_{III} = \sqrt{\frac{2}{K}}\left[\eta_j \cos\frac{i(2j+1)\pi}{2K}\right]_{i,j=0}^{K-1} ,$$

where $\eta_j = 1/\sqrt{2}$ for $j = 0$ and $j = K$ and $\eta_j = 1$ otherwise.

It is known that $\mathcal{C}_{III}$ diagonalizes $K \times K$ structured matrix functions $\mathbf{A}(z)$ that can be expressed as the sum of Toeplitz-plus-Hankel matrix functions in the following form (this fact is developed in [8, 9] in the context of constant matrices with so-called displacement structure [10]),

$$\mathbf{A}(z) = \mathbf{T}(z) + \mathbf{H}(z) + \mathbf{B}(z) , \tag{12}$$

where $\mathbf{T}(z)$ is a *symmetric* Toeplitz matrix, $\mathbf{H}(z)$ is a Hankel matrix related to $\mathbf{T}(z)$, and $\mathbf{B}(z)$ is a "border" matrix also related to $\mathbf{T}(z)$. For example, for $K = 4$, $\{\mathbf{A}(z), \mathbf{B}(z), \mathbf{H}(z)\}$ have the forms

$$\begin{aligned}
\mathbf{T}(z) &= \begin{bmatrix} t_0(z) & t_1(z) & t_2(z) & t_3(z) \\ t_1(z) & t_0(z) & t_1(z) & t_2(z) \\ t_2(z) & t_1(z) & t_0(z) & t_1(z) \\ t_3(z) & t_2(z) & t_1(z) & t_0(z) \end{bmatrix} \\
\mathbf{H}(z) &= \begin{bmatrix} t_0(z) & t_1(z) & t_2(z) & t_3(z) \\ t_1(z) & t_2(z) & t_3(z) & 0 \\ t_2(z) & t_3(z) & 0 & -t_3(z) \\ t_3(z) & 0 & -t_3(z) & -t_2(z) \end{bmatrix} \\
\mathbf{B}(z) &= \begin{bmatrix} -\frac{t_0(z)}{\sqrt{2}-2} & t_1(z) & t_2(z) & t_3(z) \\ t_1(z) & & & \\ t_2(z) & & & \\ t_3(z) & & & \end{bmatrix}(\sqrt{2}-2).
\end{aligned}$$

Returning to the Toeplitz matrix $\mathbf{P}(z)$ in (2), which arises from the representation $\mathbf{G}(z) = \mathbf{P}(z)\mathbf{Q}(z)$, we now embed it into a matrix $\mathbf{A}(z)$ that can be diagonalized by $\mathcal{C}_{III}$ (in contrast to the earlier embedding into the circulant matrix $\mathbf{C}(z)$). We do so as follows. Assume, for simplicity, that

[2]We hasten to add that the derivation applies equally well to other trigonometric transforms, such as DCT-I to DCT-IV and DST-I to DST-IV. These transforms are also known to diagonalize matrices $\mathbf{A}(z)$ of the form (12) for different choices of the Hankel and border matrices, $\mathbf{H}(z)$ and $\mathbf{B}(z)$. We omit the details for brevity.

$M = 2$. Then

$$\mathbf{P}(z) = \begin{bmatrix} g_0(z) & g_1(z) & 0 \\ 0 & g_0(z) & g_1(z) \end{bmatrix} . \qquad (13)$$

We first embed $\mathbf{P}(z)$ into a symmetric matrix $\mathbf{T}(z)$,

$$\mathbf{T}(z) \triangleq \begin{bmatrix} 0 & g_0(z) & g_1(z) & 0 & 0 \\ g_0(z) & 0 & \boxed{g_0(z)} & \boxed{g_1(z)} & \boxed{0} \\ g_1(z) & g_0(z) & \boxed{0} & \boxed{g_0(z)} & \boxed{g_1(z)} \\ 0 & g_1(z) & g_0(z) & 0 & g_0(z) \\ 0 & 0 & g_1(z) & g_0(z) & 0 \end{bmatrix} ,$$

where the framed entries correspond to $\mathbf{P}(z)$. Then the corresponding matrix $\mathbf{A}(z)$ is (we now drop the argument z from the $g_i(z)$ for compactness of notation)

$$\mathbf{A}(z) = \begin{bmatrix} 0 & 2g_0 & 2g_1 & 0 & 0 \\ 2g_0 & g_1 & \boxed{g_0} & \boxed{g_1} & \boxed{0} \\ 2g_1 & g_0 & \boxed{0} & \boxed{g_0} & \boxed{g_1} \\ 0 & g_1 & g_0 & 0 & 2g_0 \\ 0 & 0 & g_1 & 2g_0 & 0 \end{bmatrix} + \mathbf{B}(z) .$$

We can thus recover $\mathbf{P}(z)$ from $\mathbf{A}(z)$ as

$$\mathbf{P}(z) = [\mathbf{0} \;\; \mathbf{I}_M \;\; \mathbf{0}]\mathbf{A}(z) \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{2M-1} \end{bmatrix} , \qquad (14)$$

where the column dimension of the square matrix $\mathbf{A}(z)$ is $\frac{7M-4}{2}$ when M is even and $\frac{7M-3}{2}$ when M is odd. We shall denote the dimensions of $\mathbf{A}(z)$ generically by $K \times K$. The matrix $\mathbf{A}(z)$ can thus be diagonalized by $\mathcal{C}_{III}$, say

$$\mathbf{A}(z) = \mathcal{C}_{III}^T \mathbf{W}(z) \mathcal{C}_{III} , \qquad (15)$$

where $\mathbf{W}(z) = \text{diag}\{w_i(z)\}$ has now K entries. Moreover, as in (5), and for the case $M = 2$,

$$\sqrt{\frac{5}{2}} \begin{bmatrix} 0 \\ a_1(z) \\ a_2(z) \\ 0 \\ 0 \end{bmatrix} = \mathcal{C}_{III}^T \begin{bmatrix} w_0(z) \\ w_1(z) \\ w_2(z) \\ w_3(z) \\ w_4(z) \end{bmatrix} , \qquad (16)$$

where we used the fact that the top row of $\mathbf{A}(z)$ has the form $[0 \;\; a_1(z) \;\; a_2(z) \;\; 0 \;\; 0]$ for some $\{a_1(z), a_2(z)\}$.

We now have K FIR filters to adapt, with weight vectors $\{\mathbf{w}_i\}$ and regression vectors $\{\mathbf{x}_i'(n)\}$ where

$$\mathbf{x}_i'(z) = \mathcal{C}_{III} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{2M-1} \end{bmatrix} \mathbf{Q}(z)\mathbf{x}(z) .$$

If we define, as before,

$$\mathcal{W} = \begin{bmatrix} \mathbf{w}_0^T \\ \mathbf{w}_1^T \\ \vdots \\ \mathbf{w}_{K-1}^T \end{bmatrix} , \quad \mathbf{X}(n) = \begin{bmatrix} \mathbf{x}_0'(n) \\ \mathbf{x}_1'(n) \\ \vdots \\ \mathbf{x}_{K-1}'(n) \end{bmatrix} ,$$

and let $\mathbf{E}(n) = \text{diag}\{\mathbf{e}_0'(n), \ldots, \mathbf{e}_{K-1}'(n)\}$, where

$$\mathbf{e}'(n) \triangleq \mathcal{C}_{III} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_M \\ \mathbf{0} \end{bmatrix} \mathbf{e}(n) , \qquad (17)$$

then we obtain the following *constrained* adaptive version

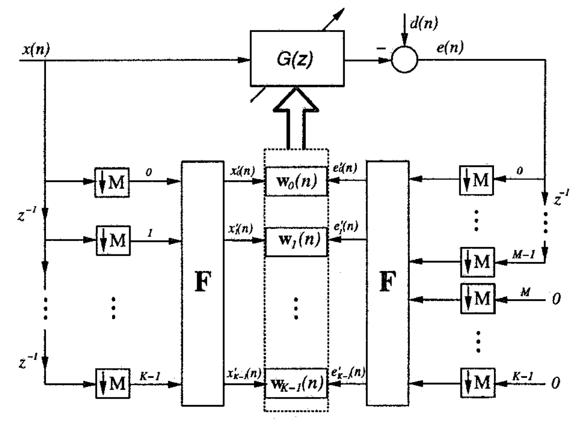
$$\mathcal{W}_{n+1}^c = \mathcal{W}_n^c + \mu \mathcal{C}_{III} \mathbf{U}_T \mathcal{C}_{III}^T \mathbf{\Lambda}^{-1}(n) \mathbf{E}(n) \mathbf{X}(n) , \qquad (18)$$

where $\mathbf{U}_T = (\mathbf{0} \oplus \mathbf{I}_M \oplus \mathbf{0})$.

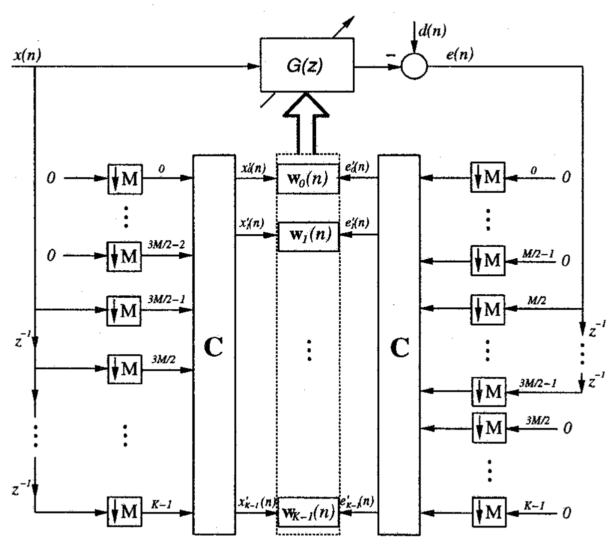
The constraints incorporated into (11) and (18) can be interpreted as a mapping of the data in the subbands into wideband by multiplication with $\mathbf{F}$ in the DFT case and with $\mathcal{C}_{III}^T$ in the DCT case. By applying the inverses of the corresponding transforms, we map the wideband filter back into the frequency domain. In the original DFT-MDF structure [2], the wideband filter is transformed back into the subband filters and convolved with the subband signals $x_i'(n)$. The output is then reconstructed and subtracted from $d(n)$. Alternatively, the convolution can be performed separately, as shown in Figs. 2(a) for the DFT-MDF and (b) for the proposed DCT-MDF. The reasons for this is that 1) it can be performed efficiently with a different block size R, optimized to reduce the computational complexity, and 2) the delay in the signal path can be eliminated if we compute the first output block by direct convolution [11]. Figure 2(c) illustrates the convolution part in (a) and (b), which is performed without delay using FFTs with a block size R.

5 Some Simulation Results

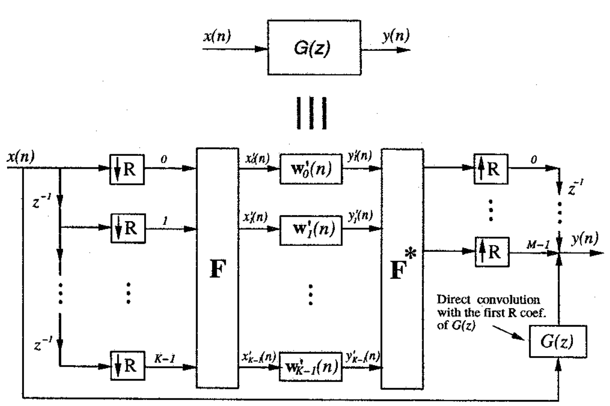
We compare the convergence performance of the DFT and DCT structures for a second-order AR input signal $x(n)$. The learning curves are shown in Fig. 3. The block sizes were adjusted for each structure so that the corresponding algorithm exhibited the best performance. The DFT-MDF was tested with a block size $M = 8$ (corresponding to subband filters of size 8 each), while the DCT-based filter had a block size $M = 64$ (corresponding to subband filters of single tap each). Note that these block sizes are only for the adaptation process, because the convolution can be performed without delay with a different optimized block size. The length of the impulse response was $N = 64$, and the convergence factor used for the adaptive algorithm was $\mu = M/N$. The DCT can be computed efficiently using the algorithm in [5], and the computational complexity involved is similar to existing frequency-domain algorithms. The convergence behavior of the proposed trigonometric-based adaptive structures is currently under investigation.



(a)



(b)



(c)

Figure 2. Delayless (a) DFT-MDF and (b) DCT-MDF for even M. (c) Delayless convolution based on the DFT.

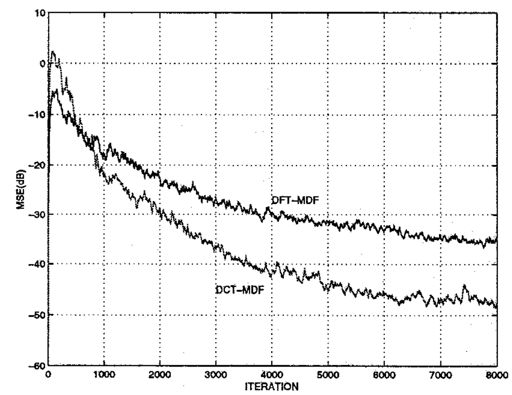


Figure 3. MSE decay for colored input signal for the DFT-MDF and the proposed DCT-MDF.

References

[1] G. A. Clark, S. K. Mitra, and S. R. Parker, "Block implementation of adaptive digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-28, pp. 584–592, 1981.

[2] K. P. J. Soo, "Multidelay block frequency domain adaptive filters," *IEEE Trans. Acoust. Speech Signal Processing*, vol. 38, no.2, pp. 373-376, 1990.

[3] J. Shynk, "Frequency-domain and multirate adaptive filtering," *Signal Processing Mag.*, vol. 1, pp. 15-37, 1992.

[4] M. R. Petraglia and S. K. Mitra, "Performance analysis of adaptive filter structures based on subband decomposition," in *Proc. ISCAS*, pp. 60-63, 1993.

[5] M. Vetterli and H. J. Nussbaumer, "Simple FFT and DCT algorithms with reduced number of operations," *Signal Processing*, pp. 267-278, 1984.

[6] V. P. Sathe and P. P. Vaidyanathan, "Effects of multirate systems on the statistical properties of random signals," *IEEE Trans. Signal Processing*, vol. 41, no. 1, pp. 131-146, Jan. 1993.

[7] I. -S. Lin and S. K. Mitra, "Overlapped block digital filtering," *IEEE Trans. Circuits Syst. II*, vol. 43, no.8, pp. 586-596, Aug. 1996.

[8] G. Heinig and K. Rost, "Representations of Toeplitz-plus-Hankel matrices using trigonometric transformations with application to fast matrix-vector multiplication," *Linear Algebra and Its Applications*, vol. 275–276, pp. 225–248, 1998.

[9] T. Kailath and V. Olshevsky, "Displacement structure approach to discrete-trigonometric-transform based preconditioners of G. Strang type and of T. Chan type," preprint.

[10] T. Kailath and A. H. Sayed, "Displacement structure: Theory and Applications," *SIAM Review*, vol. 37, no. 3, pp. 297–386, Sep. 1995.

[11] D. R. Morgan and M. J. C. Thi, "A delayless subband adaptive filter architecture," *IEEE Trans. Signal Processing*, vol. 43, pp. 1818-1830, Aug. 1995.