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Regularized Robust Filtering for Discrete Time Uncertain Time-Delayed Stochastic Systems

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The Kalman filter is the optimal linear least-mean-squares estimator for systems that are described by linear state-space Markovian models [1]. However, when the model is not accurately known, the performance of the filter can deteriorate appreciably. There have been many approaches to robust filtering in the literature (see, e.g., [2]). In [3, 4], frameworks for robust filter designs were discussed that perform regularization as opposed to de-regularization. In this paper, we pursue the design of such regularized robust filters for state-delayed systems. We also allow for stochastic uncertainties in the state matrices and deterministic uncertainties for the output matrices and design a robust filter that bounds the state error covariance matrix. Thus consider an $n-$dimensional state-space model of the form

$$\begin{aligned} x_{k+1} &= Fx_k + F_d x_{k-\tau} + Gu_k & (1)\\ y_k &= (H+\delta H_k)x_k + v_k, \quad k \geq 0 & (2)\end{aligned}$$

where $\{u_k, v_k\}$ are uncorrelated white zero-mean random processes with covariance matrices $\mathcal{E}u_k u_k^T = Q_k < \rho_u I$, $\mathcal{E}v_k v_k^T = R < \rho_v I$, and x_0 is a zero-mean random variable that is uncorrelated with $\{u_k, v_k\}$ for all k. Here, the symbol $\mathcal{E}$ denotes expectation. The uncertainties δH_k are modeled as $\delta H_k = M\Delta_k E$ where M and E are known matrices, while Δ_k is an arbitrary contraction, $\|\Delta_k\| < 1$. We shall consider two types of uncertainty descriptions for the state matrix F. One type is in terms of polytopic uncertainties and the other is in terms of stochastic uncertainties. We assume that F is described by

$$F = F_o + \delta F_k, \quad \delta F_k = N\bar{\Delta}_k J \tag{3}$$

for some known $\{N, J\}$ and where $\bar{\Delta}_k$ is a random matrix whose entries are zero mean and uncorrelated with each other, and such that $\mathcal{E}\bar{\Delta}_k\bar{\Delta}_k^T \leq \rho_{\bar{\Delta}} I$, for some known positive scalar $\rho_{\bar{\Delta}}$. Moreover, F_o lies inside a convex bounded polyhedral domain $\mathcal{K}$ that is described by m vertices as follows:

$$\mathcal{K} = \left\{ F_o = \sum_{i=1}^{m} \alpha_i F_i, \;\; \alpha_i \geq 0, \;\; \sum_{i=1}^{m} \alpha_i = 1 \right\} \tag{4}$$

Our objective is to design a robust linear estimator for the state variable x_k of the form

$$\hat{x}_{k|k} = F_p \hat{x}_{k|k-1} + K_p y_k, \qquad \hat{x}_{k+1|k} = F_c \hat{x}_{k|k} \tag{5}$$

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for some matrices F_p and K_p to be determined in order to minimize the state error covariance matrix and where F_c denotes the centroid of the polytope $\mathcal{K}_k$: $F_c = \frac{1}{m}\sum_{i=1}^{m} F_i$.

Robust filter: For positive definite matrices W, P_1 and P_2 of appropriate dimensions, and matrices S_1, S_2 and S_3 such that $I < \begin{pmatrix} S_1 & S_3 \\ S_3^T & S_2 \end{pmatrix}$, define

$$\begin{aligned} F_p &= F_c(I - \hat{\beta} W E^T E - W H^T \hat{R}^{-1} H), \quad K_p = F_c W H^T \hat{R}^{-1} \\ Q_1 &= F_p^T P_2, \quad Q_2 = K_p^T P_2 \end{aligned}$$

where $\hat{R}^{-1} = (R - \hat{\beta}^{-1} M M^T)^{-1}$ for some positive $\hat{\beta}$ chosen as explained in [3], and also define

$$X = \begin{pmatrix} B & -S_3 & \tilde{J} & 0 \\ -S_3^T & P_2 - S_2 & -Q_1 F_d & 0 \\ \tilde{J}^T & -F_d^T Q_1^T & S_1 - F_d^T (P_1 + P_2) F_d & S_3 \\ 0 & 0 & S_3^T & S_2 \end{pmatrix}, \quad Y^T = \begin{pmatrix} P_1 F & 0 & 0 & 0 \\ \hat{J}^T & Q_1^T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \quad (6)$$

where

$$\begin{aligned} B &\triangleq P_1 - \rho_\Delta J^T N^T (P_1 + P_2) N J - S_1 \\ \hat{J} &\triangleq -H^T Q_2 - Q_1 + F_c^T P_2 \\ \tilde{J} &\triangleq -F_c^T (P_1 + P_2) F_d + H^T Q_2 F_d + Q_1 F_d \end{aligned}$$

Solve the following convex optimization problem over the variables $\{P_1, P_2, Q_1, Q_2, \Lambda\}$:

$$\min \ \operatorname{Tr}(\rho_u G^T (P_1 + P_2) G + \rho_v \Lambda) \quad (7)$$

subject to conditions

$$\begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix} > \alpha I, \quad \begin{pmatrix} \Lambda & Q_2 \\ Q_2^T & P_2 \end{pmatrix} > 0 \quad (8)$$

Then set $F_p = P_2^{-1} Q_1^T$, and $K_p = P_2^{-1} Q_2^T$.

References

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