FUNDAMENTALS OF ADAPTIVE FILTERING

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Readers are welcome to bring to the attention of the author at **sayed@ee.ucla.edu** any typos or suggestions for improvements. The author is thankful for all feedback.

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ERRATA

Chapter 2

- Prob. 2.4: replace $\mathsf{E}\,\tilde{\boldsymbol{x}}W\tilde{\boldsymbol{x}}^*$ by $\mathsf{E}\,\tilde{\boldsymbol{x}}^*W\tilde{\boldsymbol{x}}$.
- Prob. 2.8: the expressions for $\bar{y}, R_y, R_{xy}$ and K_o should be replaced by:

$$R_y = \begin{bmatrix} -4p^2+4p+1 & -6p^2+6p-1 \\ -6p^2+6p-1 & -9p^2+9p \end{bmatrix},\ R_{xy} = \begin{bmatrix} -4p^2+4p & -6p^2+6p-1 \\ -4p^2+4p-1 & -6p^2+6p-1/2 \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} 2p-1 \\ \frac{3}{2}(2p-1) \end{bmatrix},\ K_o = \frac{1}{-21p^2+21p-1}\begin{bmatrix} -12p^2+12p-1 & -6p^2+6p-1 \\ 6p^2-1/2 & -14p^2+14p-3/2 \end{bmatrix}$$

- Prob. 2.17: replace $\{\boldsymbol{v},\boldsymbol{v}\}$ by $\{\boldsymbol{v},\boldsymbol{w}\}$.
- Appendix 2.A, page 104, equation (2.A.1), remove "and $B > 0$" from the left-hand side.
- Appendix 2.C, Page 112, fourth equation on page: replace $(H_i\tilde{\boldsymbol{x}}_{i|i-1} + 0$ by $(H_i\tilde{\boldsymbol{x}}_{i|i-1})^* + 0$.

Chapter 3

- Prob. 3.2, part (a): replace $c^*z = \alpha - c^*w^o$ by $c^*z = \alpha - c^*R_u^{-1}R_{du}$.
- Prob. 3.3, part (c): replace "is now given by" by "is now related to".
- Prob. 3.7, part (d): replace R_α by R_z and $\beta^* = b^*_{\text{opt}}$ by $[1\ \ \beta^*] = b^*_{\text{opt}}$.
- Prob. 3.12, Fig. 3.5: the label $\boldsymbol{y}(t)$ should appear at the output of the channel $c(t)$; remove $\boldsymbol{y}(i)$.

Chapter 4

- Page 189, last sentence of Theorem 4.4.1: replace "divergent sequence" by "divergent series."
- Prob. 4.8, part (a): replace $y \geq 0$ by $0 \leq y < 1$.
- Prob. 4.9, part (a): replace "< 1" by "≤ 1".
- Prob. 4.10, part (a): replace $2 + \epsilon/\lambda_{\max}$ by $2 + 2\epsilon/\lambda_{\max}$.
- Prob. 4.15, part (b): replace $\sigma_d = \mathsf{E}\,\boldsymbol{d}^2$ by $\sigma_d^2 = \mathsf{E}\,\boldsymbol{d}^2$.
- Prob. 4.17: replace "constant number γ" by "positive number γ".
- Prob. 4.21: replace the reference to Prob. 4.21 by a reference to Prob. 3.2.

Chapter 5

- Page 242, first line, replace "the equality $d_i = u_i w_i$" by "the equality $d_i = U_i w_i$"
- Page 245, statement of Alg. 13.2: replace $\|\tilde{u}_{i-j+k}\|^2$ by $\|\tilde{u}_{i-k+j}\|^2$ and $\tilde{u}_{i-k}$ by $\tilde{u}^*_{i-k}$.
- Prob. 5.13, page 260, top paragraph: replace $\{u_i, u_{i-2}, u_{i-3}\}$ by $\{u_i, u_{i-1}, u_{i-2}\}$.
- Prob. 5.25, part (b): replace "w_i that solves" by "w_i with smallest perturbation to w_{i-1} that solves".
- Prob. 5.26, part (b): replace last $|h(i)|^4$ in $J(w)$ by $|h(i)|^2$. Also replace "$h(i_o) = 1$" by "$|h(i_o)| = 1$".

Chapter 6

- Page 304, the expressions for $\mathsf{E}\,\boldsymbol{p}^2(i)$ and α_u should read (second terms are missing):

$$\begin{aligned} \mathsf{E}\,\boldsymbol{p}^2(i) &= (1-\beta)^2\mathsf{E}\left[\sum_{j=0}^{i}\beta^{2j}|\boldsymbol{u}(i-j)|^2 + \sum_{j_1=0}^{i}\sum_{j_2=0,j_2\neq j_1}^{i}\beta^{j_1+j_2}|\boldsymbol{u}(i-j_1)|^2\cdot|\boldsymbol{u}(i-j_2)|^2\right] \\ &= \gamma\sigma_u^4(1-\beta^{2(i+1)})\frac{1-\beta}{1+\beta} + 2\sigma_u^4(1-\beta^{i+1})(1-\beta^i)\frac{\beta}{1+\beta} \\ \alpha_u &\approx \frac{\mathsf{E}\,\|\boldsymbol{u}_i\|^2}{\mathsf{E}\,\boldsymbol{p}^2(i)} \to \frac{M(1+\beta)}{\sigma_u^2[\gamma(1-\beta)+2\beta]} \text{ as } i\to\infty \end{aligned}$$

Expression for EMSE in (6.6.14) and in Lemma 6.6.2 becomes (adjust denominator)

$$\zeta^{\epsilon-\text{pNLMS}} = \frac{\mu(1+\beta)M\sigma_v^2}{2\,[\gamma(1-\beta)+2\beta] - \mu M(1+\beta)}$$

- Page 344, part (e): replace "Use the last 5600 of the signals.." by "Use the last 5600 samples of the signals.."
- Page 350, equation (6.A.5): in the expression for $\sin^2(\theta_i)$, replace $\|\tilde{\boldsymbol{w}}_{i-1}\|^2$ by $\|\tilde{\boldsymbol{w}}_i\|^2$ here and in the expression following (6.A.3) on page 349.

Chapter 7

- Page 372, expression for α_u after (7.6.11):

$$\alpha_u \;\approx\; \frac{M(1+\beta)}{\sigma_u^2[\gamma(1-\beta)+2\beta]}$$

  Expression for EMSE in (7.6.12) and in Lemma 7.6.2 (adjust the denominator):

$$\zeta^{\epsilon-\mathsf{pNLMS}} = \frac{\mu M(1+\beta)\sigma_v^2 + \mu^{-1}\gamma\sigma_u^2(1-\beta)\mathsf{Tr}(Q)}{2\left[\gamma(1-\beta)+2\beta\right] - \mu M(1+\beta)}$$

- Page 373, equation (7.7.8): replace $\mu\mathsf{Tr}(R_u)$ by $2\mu\mathsf{Tr}(R_u)$.
- Page 387, Table 7.3, expression for EMSE of ϵ-NLMS with power normalization in the third row of the table should be replaced by the same expression shown above for page 372.
- Prob. 7.1, part (a): replace $4\eta_u^2\sigma_v^2$ by $4\eta_u^2\sigma_v^4$.
- Prob. 7.3, weight vector update: replace $\boldsymbol{U}_i w_{i-1}$ by $\boldsymbol{U}_i w_{i-1-\alpha(K-1)}$.

Chapter 8

- Prob. 8.1, part (a): the expression for α should be scaled by 1/2.

Chapter 9

- Prob. 9.3, part (b): replace "negative-definite" by "indefinite".
- Prob. 9.4: replace $1-c$ by $1-\mu c$.
- Prob. 9.15, part (b): replace $\|\overline{\boldsymbol{u}}_i\|^2_{\overline{\Sigma}}$ by $\|\check{\boldsymbol{u}}_i\|^2_{\check{U}\overline{\Sigma}\check{U}^*}$. In the expression of part (d), remove the factor 2.
- Prob. 9.30, part (b): expression for D should be $D = 2\mu\Lambda^\alpha - \mu^2(\Lambda^\alpha)^2$.
- Probs. 9.32 and 9.33: it is assumed in these problems that the nonstationary model is $\boldsymbol{w}_i^o = w^o + \boldsymbol{q}_i$ (i.e., it consists of random perturbations around a constant w^o) rather than as in item (2) of Prob. 9.31.
- Prob. 9.33: replace σ_v^2 by $\sigma_{\bar{v}}^2$.

Chapter 10

- Prob. 10.5: the identity should read as follows: $u(i) * [e^{j\omega_o i}h(i)] = e^{j\omega_o i}\left([e^{-j\omega_o i}u(i)] * h(i)\right)$.
- Prob. 10.9: first row of C should be divided by $\sqrt{K}$ and not K.
- Prob. 10.11: replace $1/(M-1)$ by $1/M$.
- Last equation on page 607: rightmost term should be $e^{j\omega_k i}\left([e^{-j\omega_k i}u(i)] * h(i)\right)$.
- Page 609: replace $R(ze^{j2\pi k/K})$ and $R(e^{j(\omega+\omega_k)})$ by $R(ze^{-j2\pi k/K})$ and $R(e^{j(\omega-\omega_k)})$. Also, $r_k(i) = e^{j\omega_k i}r(i)$ and the last equation on the page should be $s'_k(i) * r_k(i) = s'_k(i) * e^{j\omega_k i}r(i)$.

Chapter 11

- Page 674, footnote 8, next to last line, replace $\mathcal{R}(y)$ by $\mathcal{R}(H)$.
- Page 682, expression for P_z in the middle of the page after (11.5.38): the locations of the upper and lower triangular matrices involving $\widehat{w}^b$ should be exchanged.
- Page 683, second equality in the expression for γ_z at the bottom of the page: same as above.
- Prob. 11.6, replace $\|d_i - U_i w_{i-1}\|^2$ by $\|d_i - U_i w_i\|^2$.
- Prob. 11.13, part (a): replace (11.5.1) by (11.5.7). Also, replace "$\widehat{w} =$" by "$w_s =$".
- Prob. 11.16, part (a): replace "$\widehat{w} =$" by "$w_s =$" and remove third line of [0 1] on the right.
- Eq. (11.9.11): replace $+b_2(i)$ by $-b_2(i)$.
- Prob. 11.28, part (b): ignore the correlations between $\{s_1, s_2\}$ and their shifted versions.

Chapter 12

- Page 741, right above (12.4.1): replace $(N-1)\times M$ by $N\times M$.
- Prob. 12.3: replace $P_{-1} = \Pi$ by $P_{-1} = \Pi^{-1}$.

- Prob. 12.5, part (a): w should multiply col$\{R_{N-1}, u_N\}$ on the right-hand side.
- Prob. 12.8, part (a): remove the * from the second U_N in recursion for P_N. In part (c), $w_{-1} = \bar{w}$.

Chapter 13

- Page 779, first sentence in Lemma 13.3.1: replace "Given" by "Consider".
- Page 810, second paragraph: replace z_o by $z(0)$.
- Prob. 13.10, part (a): replace f^2 by a^2.

Chapter 14

- Page 822: in the second and third equations after (14.1.13), the quantities C and D should be replaced by D and E, respectively.
- Alg. 14.6.1 and Prob. 14.5: replace ξ by ζ. Also, in Alg. 14.6.1, $\gamma_M^{-1}(i) = \zeta_M^f(i)\zeta_M^{-b}(i)/\lambda^M$.
- Prob. 14.10: it is assumed that $u_i\Psi = u_{i-1}$.
- Prob. 14.11, downdating step: replace $g_{i-1}^L \gamma_L^{-1/2}(i-1)$ by $-g_{i-1}^L \gamma_L^{-1/2}(i-1)$.
- Prob. 14.14, part (a): 2nd column of pre-array should read as follows:

$$\begin{bmatrix} \begin{bmatrix} u^{(1)}(i) & u_{i-1}^{(1)} & \dots & u^{(N)}(i) & u_{i-1}^{(N)} \end{bmatrix} \bar{L}_{i-1} \\ \bar{L}_{i-1} \end{bmatrix}, \quad \text{where} \quad \bar{L}_{i-1} = \text{diag}\left\{\bar{L}_{i-1}^{(1)}, \dots, \bar{L}_{i-1}^{(N)}\right\}$$

- Prob. 14.15: definitions should read as follows:

$$u_i^{(3)} = \begin{bmatrix} u(i)u(i-1) & \dots & u(i-M+2)u(i-M+1) \end{bmatrix}$$

$$u_i^{(4)} = \begin{bmatrix} u(i)u(i-2) & \dots & u(i-M+3)u(i-M+1) \end{bmatrix}, \quad u_i^{(M+2)} = \begin{bmatrix} u(i)u(i-M+1) \end{bmatrix}$$

$$w^{o(3)} = \begin{bmatrix} w^o(0,1) & \dots & w^o(M-2,M-1) \end{bmatrix}$$

$$w^{o(4)} = \begin{bmatrix} w^o(0,2) & \dots & w^o(M-3,M-1) \end{bmatrix}, \quad w^{o(M+2)} = \begin{bmatrix} w^o(0,M-1) \end{bmatrix}$$

Chapter 15

- Page 881, equation (15.3.4), replace $\lambda^i w^{\bar{b}*}\Pi_2 w^{\bar{b}}$ by $\lambda^i w_2^{\bar{b}*}\Pi_2 w_2^{\bar{b}}$.
- Page 898, Figure 15.6: the time index for the reflection coefficients in the figure should be $(i-1)$ instead of (i).
- Page 914, second equation: replace $\bar{f}_M(i)$ by $\bar{b}_M(i)$.
- Page 915, Algorithm 15.12.1, step 1: add the initial condition $\zeta_m^{b/2}(-2) = \sqrt{\eta^{-1}\lambda^{-m-3}}$.
- Prob. 15.4: replace $\zeta_m^{b/2}(i-1)$ by $\zeta_m^b(i-1)$ in expression for ρ.
- Prob. 15.5: add under initial conditions $c_m^b(-1) = 1$, $s_m^b(-1) = 0$, $b'_m(-1) = 0$.
- Prob. 15.9: in the state estimator equations of parts (a), (b), and (c), a factor of $\lambda^{1/2}$ should multiply $p^{\bar{b}}(i+1|i)$, $p^f(i+1|i)$, and $p(i+1|i)$.

Chapter 16

- Eq. (16.4.24): replace $u(i,M)$ by $u(i-1,M-1)$.
- Prob. 16.6: a matrix Ψ should multiply the first two terms on the right-hand side from the left.
- Prob. 16.7: the reference is to Prob. 14.2. Also, replace $\beta_M(i)$ by $\breve{\beta}_M(i)$.
- Prob. 16.12: replace ξ_M^l by $\xi_M^l(i)$. Also, in part (b), replace a by $|a|$.
- Prob. 16.16: replace $\mathcal{V}_k(z)$ inside the sum by $\mathcal{V}_m(z)$.

Chapter 17

- Eq. (17.2.18): replace P_N by P_{N-1}.
- Prob. 17.3, recursion for w_i: replace α by α^{-1}.
- Prob. 17.5, part (a): remove ϵ from the denominator.
- Prob. 17.13, part (d): replace $\|u_i\|^2 > \epsilon > 0$ by $\|u_i\|^{-2} > \epsilon > 0$. Also, replace $w_i \to w$ by $w_i \to w^o$.
- Prob. 17.14, part (c), rephrase as follows: "Follow arguments similar to part (d) of Prob. 17.13 to conclude that $w_i \to w^o$.
- Prob. 17.15, part (b): replace μ by $\mu(i)$ inside the boxed expression.
- Prob. 17.21, part (b): min should be max.
- Prob. 17.22: in the definition of $J(w)$, remove the right-most equality that involves $v(i)$.
- Prob. 17.24, part (c), m should be M.