PREFACE

One of the most significant accomplishments in control theory during the 1980's and early 1990's has been the development of H^∞ control. Originally introduced by G. Zames in 1980, H^∞ theory is concerned with the design of controllers that are robust with respect to model uncertainty and lack of statistical knowledge on the exogenous signals. In this sense, it can be considered as an outgrowth and extension of the now classical LQG theory, developed in the 1950's and 1960's, which assumed perfect models and complete statistical knowledge. Whereas such assumptions were reasonable for problems in the guidance and manuevering of space vehicles, to which these theories were first applied, they were much less so in several more mundane industrial problems. Although other approaches to robust control have also been introduced and studied, H^∞ theory has by far attracted the most attention and has been developed by many authors using various ingenious methods and employing tools from interpolation-theory, operator-theory, game-theory, circuit-theory, and system-theory. The above abundance of viewpoints poses at least two challenges: one practical, how to master them all? And one theoretical: what is the theoretical core of H^∞ theory? Rather than attempting to address these questions directly, the theme of this monograph is that such very different solution methods need not be necessary; the basic LQG and Kalman filtering arguments can still be used, provided we set up appropriate control and estimation problems with elements not in a Hilbert space, but in an indefinite metric (so-called Krein) space.

Although Hilbert spaces and Krein spaces share many characteristics, they differ in special ways that turn out to mark the differences between the stochastic LQG (or H^2) theories and the more recent deterministic H^∞ theories. The fact that indefinite metric spaces enter into H^∞ theory has been noticed before (*e.g.*, in studies using what is called J-spectral facorization methods), but the simple and complete explanation of certain formal similarities, as well as certain puzzling differences, between LQG and H^∞ results that we give in this book does not seem to have been recognized before.

The Krein space formulation presented in this book provides a unified approach for problems in both LQG and H^∞ (risk-sensitive and game-theoretic) estimation and control. Most importantly, it allows one to use the insight obtained from over three decades of work in traditional LQG theory to obtain new results in the other areas. Proceeding in this spirit, this book generalizes the conventional array algorithms of H^2 estimation and control to the H^∞ setting, and presents new results on the asymptotic behavior of H^∞ filters and controllers, and on the existence and properties of solutions of Riccati equations with (possibly) indefinite coefficient matrices.

We should also mention that our emphasis is deliberately first on estimation problems, knowing which we can then solve all the relevant control problems in detail. The philosophy of H^∞ estimation theory, and especially the tools, have ramifications for problems beyond those of control. We illustrate this by also studying several problems in adaptive filtering, signal processing, and communications from this viewpoint.

Portions of this monograph have been used in a one-quarter graduate-level course on advanced topics in estimation, control, and signal processing at Stanford. Apart from a complete reading, the following subsets can be independently used to cover more specific topics:

- **Finite-Horizon Problems:** Chapters 1 to 7 for estimation problems, and Chapters 8 and 9 for control.
- **Infinite-Horizon Problems:** Chapters 1, and 10 to 14.
- **State-Space Methods:** Chapters 1 to 9 and 12 to 14.
- **Input-Output Methods:** Chapters 10, 11, and 15.
- **Estimation:** Chapters 1 to 6 and Chapters 10 and 13.
- **Control:** Chapters 1, 8, 9, 11, and 13.
- **Applications:** Chapters 7 and 15.
- **Continuous-Time Results:** Chapter 16.

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To the memory of

GEORGE ZAMES

innovator and pioneer,
gentleman and friend.