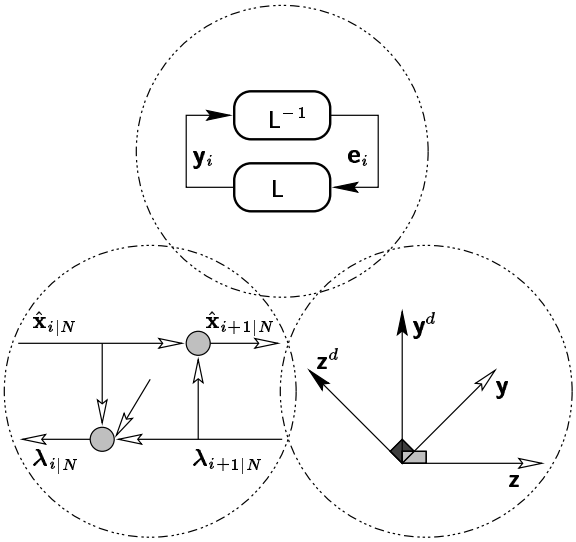
LINEAR ESTIMATION



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PREFACE

The problem of estimating the values of a random (or stochastic) process given observations of a related random process is encountered in many areas of science and engineering, *e.g.,* communications, control, signal processing, geophysics, econometrics and statistics. Although the topic has a rich history, and its formative stages can be attributed to illustrious investigators such as Laplace, Gauss, Legendre, and others, the current high interest in such problems began with the work of H. Wold, A. N. Kolmogorov, and N. Wiener in the late 1930s and early 1940s. N. Wiener in particular stressed the importance of modeling not just "noise" but also "signals" as random processes. His thought-provoking originally classified 1942 report, released for open publication in 1949 and now available in paperback form under the title *Time Series Analysis*, is still very worthwhile background reading.

As with all deep subjects, the extensions of these results have been very far-reaching as well. A particularly important development arose from the incorporation into the theory of multichannel state-space models. Though there were various earlier partial intimations and explorations, especially in the work of R. L. Stratonovich in the former Soviet Union, the chief credit for the explosion of activity in this direction goes to R. E. Kalman, who also made important related contributions to linear systems, optimal control, passive systems, stability theory, and network synthesis.

In fact, least-squares estimation is one of those happy subjects that is interesting not only in the richness and scope of its results, but also because of its mutually beneficial connections with a host of other (often apparently very different) subjects. Thus, beyond those already named, we may mention connections with radiative transfer and scattering theory, linear algebra, matrix and operator theory, orthogonal polynomials, moment problems, inverse scattering problems, interpolation theory, decoding of Reed–Solomon and BCH codes, polynomial factorization and root distribution problems, digital filtering, spectral analysis, signal detection, martingale theory, the so-called $\mathcal{H}_\infty$ theories of estimation and control, least-squares and adaptive filtering problems, and many others. We can surely apply to it the lines written by William Shakespeare about another (beautiful) subject:

> "Age does not wither her, nor custom stale,
> Her infinite variety."

Though we were originally tempted to cover a wider range, many reasons have led us to focus this volume largely on estimation problems for finite-dimensional linear systems with state-space models, covering most aspects of an area now generally known as Wiener and Kalman filtering theory. Three distinctive features of our treatment are the pervasive use of a geometric point of view, the emphasis on the numerically favored square-root/array forms of many algorithms, and the emphasis on equivalence and duality concepts for the solution of several related problems in adaptive filtering, estimation, and control. These features are generally absent in most prior treatments, ostensibly on the grounds that they are too abstract and complicated. It is our hope that these misconceptions will be dispelled by the presentation herein, and that the fundamental simplicity and power of these ideas will be more widely recognized and exploited.

The material presented in this book can be broadly categorized into the following areas:

- Introduction and Foundations

  Chapter 1: Overview
  Chapter 2: Deterministic Least-Squares Problems
  Chapter 3: Stochastic Least-Squares Problems
  Chapter 4: The Innovations Process
  Chapter 5: State-Space Models

- Estimation of Stationary Processes

  Chapter 6: Innovations for Stationary Processes
  Chapter 7: Wiener Theory for Scalar Processes
  Chapter 8: Recursive Wiener Filters

- Estimation of Nonstationary Processes

  Chapter 9: The Kalman Filter
  Chapter 10: Smoothed Estimators

- Fast and Array Algorithms

  Chapter 11: Fast Algorithms
  Chapter 12: Array Algorithms
  Chapter 13: Fast Array Algorithms

- Continuous-Time Estimation

  Chapter 16: Continuous-Time State-Space Estimation

- Advanced Topics

Chapter 14: Asymptotic Behavior

Chapter 15: Duality and Equivalence in Estimation and Control

Chapter 17: A Scattering Theory Approach

Being intended for a graduate-level course, the book assumes familiarity with basic concepts from matrix theory, linear algebra, linear system theory, and random processes. Four appendices at the end of the book provide the reader with background material in all these areas.

There is ample material in this book for the instructor to fashion a course to his or her needs and tastes. The authors have used portions of this book as the basis for one-quarter first-year graduate level courses at Stanford University, the University of California at Los Angeles, and the University of California at Santa Barbara; the students were expected to have had some exposure to discrete-time and state-space theory. A typical course would start with Secs. 1.1–1.2 as an overview (perhaps omitting the matrix derivations), with the rest of Ch. 1 left for a quick reading (and re-reading from time to time), most of Chs. 2 and 3 (focusing on the geometric approach) on the basic deterministic and stochastic least-squares problems, Ch. 4 on the innovations process, Secs. 6.4–6.5 and 7.3–7.7 on scalar Wiener filtering, Secs. 9.1–9.3, 9.5, and 9.7 on Kalman filtering, Secs. 10.1–10.2 as an introduction to smoothing, Secs. 12.1–12.5 and 13.1–13.4 on array algorithms, and Secs. 16.1–16.4 and 16.6 on continuous-time problems.

More advanced students and researchers would pursue selections of material from Sec. 2.8, Chs. 8, 11, 14, 15, and 17, and Apps. E and F. These cover, among other topics, least-squares problems with uncertain data, the problem of canonical spectral factorization, convergence of the Kalman filter, the algebraic Riccati equation, duality, backwards-time and complementary models, scattering, etc. Those wishing to go on to the more recent $\mathcal{H}_\infty$ theory can find a treatment closely related to the philosophy of the current book (*cf.* Sec. 1.6) in the research monograph of Hassibi, Sayed, and Kailath (1999).

A feature of the book is a collection of nearly 300 problems, several of which complement the text and present additional results and insights. However, there is little discussion of real applications or of the error and sensitivity analyses required for them. The main issue in applications is constructing an appropriate model, or actually a set of models, which are further analyzed and then refined by using the results and algorithms presented in this book. Developing good models and analyzing them effectively requires not only a good appreciation of the actual application, but also a good understanding of the theory, at both an analytical and an intuitive level. It is the latter that we have tried to achieve here; examples of successful applications have to be sought in the literature, and some references are provided to this end.

Acknowledgments

The development of this textbook has spanned many years. So the material, as well as its presentation, has benefited greatly from the inputs of the many bright students who have worked with us on these topics: J. Omura, P. Frost, T. Duncan, R. Geesey, D. Duttweiler, H. Aasnaes, M. Gevers, H. Weinert, A. Segall, M. Morf, B. Dickinson, G. Sidhu, B. Friedlander, A. Vieira, S. Kung, B. Levy, G. Verghese, D. Lee, J. Delosme, B. Porat, H. Lev-Ari, J. Cioffi, A. Bruckstein, T. Citron, Y. Bresler, R. Roy, J. Chun, D. Slock, D. Pal, G. Xu, R. Ackner, Y. Cho, P. Park, T. Boros, A. Erdogan, B. Halder, and our young friend Amir Ghazanfarian from whom we had so much more to learn. We are also grateful to U. Forsell, H. Hindi, V. Nascimento, T. Pare, and R. Merched for assistance and suggestions with revisions of the various drafts.

We are of course also deeply indebted to the many researchers and authors in this beautiful field. Partial acknowledgment is made through the citations and references; while the list of the latter is quite long, we apologize for omissions and inadequacies arising from the limitations of our knowledge and our energy. Nevertheless, we would indeed be remiss not to explicitly mention the inspiration and pleasure we have gained from studying the books and papers of N. Wiener, R. E. Kalman, and P. Whittle.

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