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ADAPTIVE SIGNAL PROCESSING

Signal processing is a rich discipline that deals with the extraction of information from signals. The devices that perform this task can be physical hardware devices, specialized software codes, or combinations of both. In recent years, the complexity of these devices, and the scope of their applications, have increased very dramatically with the rapidly falling costs of hardware and software. This trend has made it possible to pursue sophisticated signal processing designs at relatively low cost. Some notable applications include the suppression of interference arising from noisy measurement sensors, the elimination of distortions introduced when signals travel through transmission channels, and the recovery of signals embedded in a multitude of echoes created by multi-path effects in mobile communications.

Regardless of the application, any functional system is expected to meet certain performance specifications. The requirements, as well as the design methodology, vary according to the nature of the end application. One distinctive design methodology that has dominated much of the earlier work in the information sciences, especially in the fifties and sixties, is one that is based on statistical considerations. This framework assumes the availability of a priori information about the statistical nature of the signals involved, and then proceeds to design systems that optimize some statistical criterion. The resulting optimal designs are, in general, complex to implement and only in special, yet important, cases have they led to successful breakthroughs culminating with what are now known as the Wiener and Kalman filters.

Moreover, in many situations, a design that is motivated by statistical considerations may not be immediately feasible. This is because complete knowledge of the necessary statistical information may not be available. It may even happen that the statistical conditions vary with time. It is therefore not surprising to expect, in these scenarios, that the performance of any statistically-based optimal design will degrade, the more so as the real physical application distances itself from the modeling assumptions.

Adaptive systems provide an attractive solution to the problem. They are prominent examples of devices that adjust themselves to an ever changing environment; the structure of an adaptive system changes in such a way that its performance improves through a continuing interaction with its surroundings. Its superior performance in non-stationary environments results from its ability to track slow variations in the statistics of the signals and to continually seek optimal designs.

Adaptive signal processing is the discipline that deals with the design of adaptive systems for signal processing applications. Related issues arise in control design, where one tries to alter the behavior of a system, and lead to the study of adaptive control strategies; the main issue here is the stability of the system under feedback.

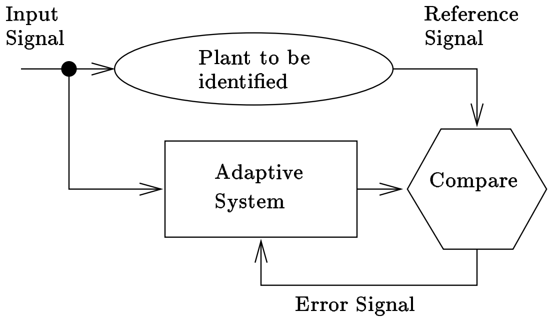

Fig. 1. Adaptive system identification.

The operation of an adaptive system can be best illustrated by considering a classical example in system identification. Figure 1 shows a plant whose input-output behavior is unknown; it may even be time-variant. The objective is to design an adaptive system that provides a good approximation to the input-output map of the plant. For this purpose, the plant is excited by a known input signal and the response is taken as a reference signal. Moreover, a structure is chosen for the adaptive system, say a finite-impulse response (FIR) structure of adequate length, and it is excited by the same input signal as the plant. At each time instant, the output of the FIR system is compared with the reference signal and the resulting error signal is used to change the coefficients of the FIR configuration. This learning process is continued over time, and one expects the output of the adaptive system to provide better tracking of the plant output as time progresses, especially when the structure of the plant is time-invariant or only slowly varying with time.

Apart from emphasizing one particular application for adaptive systems, the above example also highlights several characteristics that are common to most adaptive designs:

◇ An adaptive system adjusts itself automatically to a changing environment. This is achieved by changing the parameters of its internal structure in response to an error measure. Several adaptive structures have been used in practice but the most frequent ones are linear combiners, FIR filters, infinite-impulse response (IIR) filters, and linear combiners that are followed by nonlinearities such as sigmoid functions or nonlinear decision devices.

◇ An adaptive system interacts with its environment through an input signal and a reference signal. The reference signal is used to evaluate the performance of the adaptation process through the computation of an error signal. The adaptive system responds to the error signal in such a way so as to minimize a pre-determined cost function that is computed either from the error signal directly or from a filtered version of it.

◇ An adaptive system is inherently nonlinear and time-variant. For this reason, it is in general considerably more difficult to analyze its performance than of a linear time-invariant system. Nevertheless, adaptive systems offer more possibilities than conventional non-adaptive designs and many analysis methods have been developed that offer reasonable approximate methods for performance evaluation.

◇ An adaptive system can be trained to perform certain tasks. This usually involves a learning phase where the adaptive system is exposed to typical input and reference data and is left to adjust itself to them. At the end of the learning procedure, the system can be exposed to new data and will be expected to provide a reasonable response. Typical examples of this scenario abound in neural network applications and in the equalization of communication channels.

The performance of an adaptive system is critically dependent not only on its internal structure, but also on the algorithm used to automatically adjust the parameters that define the structure. It then comes at no surprise to know that over the years, a large variety of algorithms has been developed for this purpose. In general, the more complex the internal structure of the adaptive system, the more complex the algorithm that is needed for its adaptation. Moreover, several algorithms may exist for the same adaptive structure and the choice of which algorithm to use is dictated by several factors that include the following:

- How fast the algorithm learns in a stationary environment. That is, how fast the algorithm converges to the optimal statistical solution under stationarity assumptions. This factor determines the convergence rate of the algorithm.
- How fast the algorithm tracks statistical variations. This factor determines the tracking capability of an algorithm and is a major motivation for the use of adaptive schemes, especially in recent mobile communications applications where adaptive equalizers are used to keep track of time-variant transmission channels.
- How the performance of an adaptive scheme, operating in steady-state conditions, deviates from the performance of a statistically optimal design. This factor measures the socalled misadjustment of an adaptive scheme and serves to compare its performance with that of an optimal design in a statistical framework.
- How much computational effort is required to carry out each adjustment of the parameters of the adaptive system structure. Applications that require a large number of adaptive parameters tend to dictate a preference for computationally fast algorithms at the expense of other performance factors.
- How reliable an algorithm is in finite precision implementations. This factor is concerned with the numerical behavior of an algorithm when implemented in finite precision arithmetic, and whether numerical effects might lead to erroneous behavior.
- How robust an adaptive system is to disturbances and unmodeled dynamics. This factor determines whether small disturbances can result in large errors and therefore compromise the performance of an adaptive scheme.

The above factors are often competing requirements and it is usually necessary to seek a compromise that best suits a particular application.

There have been many procedures for the derivation of adaptive algorithms but the most frequent, at least in the sense of having had the most applications, are procedures that are based either on the method of stochastic approximation (SA) or on the least-squares (LS) criterion. In both cases, and especially for FIR adaptive structures, each criterion has led to several different variants that address in one way or another the above performance factors:

◇ In SA algorithms, a recursive procedure is devised for the minimization of a pre-determined cost function by iteratively approximating its minimum through a gradient descent procedure. While this class of algorithms generally suffers from slow convergence rates, it still enjoys widespread use due to its simplicity, low computational requirements, and often observed robustness under different operating conditions. The most prominent member of the SA algorithms is the least-mean squares (LMS) algorithm, which is undoubtedly the most widely used algorithm in current practice. Other members include filtered-error variants that are useful in active noise control and IIR system identification problems, as well as several frequency-domain adaptive schemes.

◇ In LS algorithms, a recursive procedure is devised for the minimization of a quadratic cost function. This family of algorithms is based on the least-squares criterion, which was perhaps first developed by Gauss (ca. 1795) in his work on celestial mechanics. Since then, the LS criterion has enjoyed widespread popularity in many diverse areas as a result of its attractive computational and statistical properties. Among these attractive properties, the most notable are that, for linear data models, least-squares solutions can be explicitly evaluated in closed forms, can be recursively updated as more input data is made available, and are also optimal maximum likelihood estimators in the presence of Gaussian measurement noise.

Many recursive least-squares (RLS) algorithms have been presented in the literature. Several of them are computationally more demanding than LMS type algorithms, but variants exist that are computationally competitive albeit more complex. They have better convergence properties but are less robust to disturbances.

Following a trend initiated in the late 1960's in the Kalman filtering literature, LS adaptive schemes are currently more often implemented in convenient so-called array algorithms. These algorithms are closely related to the QR method of solving systems of linear equations, and have the properties of better conditioning, reduced dynamical range, and the use of orthogonal transformations, which typically lead to better numerical performance in finite precision arithmetic. In the array form, an algorithm is described as a sequence of elementary operations on arrays of numbers. Usually, a pre-array of numbers has to be triangularized by a rotation, or a sequence of elementary rotations, in order to yield a post-array of numbers. The quantities needed to form the next pre-array can then be read off from the entries of the post-array, and the procedure can be repeated. The explicit forms of the rotation matrices are not needed in most cases. Such array descriptions are more truly algorithms in the sense that they operate on sets of numbers and provide other sets of numbers, with no explicit equations involved.

Finally, it goes without saying that, in recent years, the complexity of the varied applications for adaptive filters has increased rather dramatically and proportionally, to say the least, with the continuing advancement in ASIC (Application Specific Integrated Circuit) and sensor technologies. These technological advances have spurred an increasing range of new applications ranging from biomedical engineering to wireless communications. In these different areas, it is becoming increasingly more relevant to design adaptive schemes that exhibit a certain tolerance to overlooked factors and effects, such as unmodeled dynamics, measurement noise, and quantization errors. The field is very active and developments in related areas, such as robust and adaptive control, have furthered progress in adaptive signal processing and new insights into the study of adaptive schemes.

For more comprehensive treatments of adaptive signal processing theory and practice, the reader may consult any of the following textbook references:

◇ S. Haykin, *Adaptive Filter Theory*, 3rd edition, Prentice Hall, NJ, 1996.

◇ J. G. Proakis, C. M. Rader, F. Ling, and C. L. Nikias, *Advanced Digital Signal Processing*, Macmillan Publishing Co., NY, 1992.

◇ B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice-Hall, NJ, 1985.